

# Determination of the neutrino mass by electron capture in $^{163}\text{Ho}$ and the role of the three-hole states in $^{163}\text{Dy}$ .

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## Abstract

$^{163}\text{Ho}$  to  $^{163}\text{Dy}$  is probably due to the small  $Q$  value of about 2.5 keV the best case to determine the neutrino mass by electron capture. The energy of the  $Q$  value is distributed between the excitation of Dysprosium (and the neglected small recoil of Holmium) and the relativistic energy of the emitted neutrino including the rest mass. The reduction of the upper end of the deexcitation spectrum of Dysprosium below the  $Q$  value allows to determine the neutrino mass. The excitation of Dysprosium can be calculated in the sudden approximation of the overlap of the electron wave functions of Holmium minus the captured electron and one-, two-, three- and multiple hole-excitations in Dysprosium. Robertson [R. G. H. Robertson, Phys. Rev. C91, 035504 (2015) and arXiv: 1411.2906] and Faessler and Simkovic [Amand Faessler, Fedor Simkovic, accepted for Phys. Rev. C, March 2015 and arXiv: 1501.04338] have calculated the influence of the two-hole states on the Dysprosium spectrum. Here for the first time the influence of the three-hole states on the deexcitation bolometer spectrum of  $^{163}\text{Dy}$  is presented. The electron wave functions and the overlaps are calculated selfconsistently in a fully relativistic and antisymmetrized Dirac-Hartree-Fock approach in Holmium and in Dysprosium. The electron orbitals in Dy are determined including the one-hole states in the selfconsistent iteration. The influence of the three-hole states on the Dy deexcitation (by X-rays and Auger electrons) can hardly be seen. The three-hole states seem not to be relevant for the determination of the electron neutrino mass.

## 1 Introduction

One of the most pressing problems in particle physics is the determination of the absolute value of the neutrino mass. The single beta decay, specifically the Triton decay, can give the electron antineutrino mass [1] (present upper limit [2]:  $m_{\bar{\nu}} \leq 2.2 \text{ eV}$ ), while the neutrinoless double beta decay yields the mass of the effective Majorana electron neutrino mass [3], if the transition matrix element can be

calculated reliably. At the moment the upper limit for the Majorana neutrino mass from the double beta is about 0.3 eV.

The electron neutrino mass can be determined by electron capture. The most favorable case due to the small Q value of about 2.5 keV seems to be capture in  $^{163}\text{Ho}$  ( $Z=67$ ) to atomic excited states in  $^{163}\text{Dy}$  ( $Z=66$ ). The energy of this Q value is divided into the excitation of the Dysprosium atom, (plus the recoil of the Holmium atom, which can be neglected,) and into the energy carried away by the electron neutrino (kinetic energy and rest mass). The deexcitation spectra (X rays and Auger electrons) of all excited states in the Dysprosium atom have the upper end at the Q value minus the neutrino rest mass. The determination of the electron neutrino mass from electron capture is more complicated than the determination of the antineutrino mass from the Triton decay, since more parameters have to be fitted to the data at the upper end of the spectrum. Under the assumption, that one excited Dy resonance with a Lorentzian line shape determines the behavior at the upper end, one has to fit simultaneously four parameters to the experiment after the detector response is folded into the theoretical spectrum: (1) The neutrino mass, (2) the energy distance of the leading resonance to the Q value, (3) the width and (4) the strength of the resonance. Originally only one-hole excitations in the Dysprosium atom have been included in the theoretical determination of the spectrum [4, 5]. Recently Robertson [6] and with major improvements Faessler and Simkovic [7] have included two-hole excitations in Dysprosium. The main improvement of ref. [7] over ref. [6] was, that the electron wave functions for Holmium and for Dysprosium have been calculated in these nuclei ( $Z=67$  and  $Z=66$ ) selfconsistently in a fully relativistic and antisymmetrized Dirac-Hartree-Fock approach [8, 9, 10] and not in Xenon ( $Z=54$ ) as by Carlson and Nestor [11, 12], results used by Robertson [6]. Due to the additional occupied states in Holmium and Dysprosium more two-hole states are allowed. In addition the electron orbitals in Dysprosium are determined selfconsistently with the appropriate hole [7]. The finite size of the nuclear charge distribution is included using the Fermi parametrization determined by electron-nucleus scattering. The theoretical formulation was derived in second quantization including automatically the antisymmetrization and the overlap and exchange corrections.

In the present work this formulation is extended for the first time to the three-hole states in Dysprosium including again all the improvements of the description of the two-hole states of ref. [7].

## 2 Description of Electron Capture and the Excitation of the Three-Hole States.

The calorimetric spectrum of the deexcitation of  $^{163}\text{Dy}$  after electron capture in  $^{163}\text{Ho}$  can be expressed according to refs. [4] and [5, 7]:

$$\frac{d\Gamma}{dE_c} \propto \sum_{i=1, \dots, N_\nu} (Q - E_c) \cdot U_{e,i}^2 \cdot \sqrt{(Q - E_c)^2 - m_{\nu,i}^2} \quad (1)$$

$$\sum_{h'=b', b'(p^{-1'}q), b'(p_1^{-1}q_1')(p_2^{-1}q_2')} \lambda_0 B_{h'} \frac{\Gamma_{h'}}{2\pi} \frac{1}{(E_c - E_{h'})^2 + \Gamma_{h'}^2/4} \quad (2)$$

The second sum over the excited states  $h'$  in Dy includes the one-hole states  $b'$ , the two-hole states  $b'(p^{-1'}q)$  and the three-hole states  $b'(p_1^{-1}q_1')(p_2^{-1}q_2')$ . With the Q value  $Q = 2.3 \text{ to } 2.8 \text{ keV}$

[13, 14, 15, 16, 17] and a recommended value [18]  $Q = (2.55 \pm 0.016)$  keV. The ECHo collaboration [16] measured for the  $Q$  value for electron capture in  $^{163}\text{Ho}$  to  $^{163}\text{Dy}$ :

$$Q(\text{ECHo}) = 2.80 \pm 0.08 \text{ keV}. \quad (3)$$

$U_{e,i}^2$  is the probability for the admixture of different neutrino mass eigenstates  $i = 1, \dots, N_\nu$  into the electron neutrino and  $E_c$  is the excitation energy of final Dysprosium.  $B_b$  with the same orbital quantum numbers  $b = b'$  in Ho and Dy are the overlap and exchange corrections [5];  $\lambda_0$  contains the nuclear matrix element squared [21];  $E_{h'}$  are the one-, two- and three-hole excitation energies in Dysprosium.  $\Gamma_{h'}$  is their widths [5].

We assume, that the total atomic wave function can be described by a single Slater determinant.  $B_b$  takes into account the overlap and the exchange terms between the parent  $|G\rangle$  and the daughter atom in the state  $|A'_{b'}\rangle$  with a hole in the electron state  $|b'\rangle$ . We use the sudden approximation as Faessler et al. [22].  $B_b$  for the electron capture probability from the state  $b$  relative to the capture from  $3s_{1/2}$  with one hole in  $b'$  in the Dysprosium atom is given in eq. (4) in the Vatai approximation [23, 24]. But the numerical value used here are calculated with the full overlap and exchange corrections of Faessler et al. [5].

$$B_b = \frac{|\psi_b(R) \langle A'_{b'} | a_b | G \rangle|^2}{|\psi_{3s_{1/2}}(R)|^2} = P_b \cdot \frac{|\psi_b(R)|^2}{|\psi_{3s_{1/2}}(R)|^2} \quad (4)$$

For two-hole final states one has to multiply eq.(4) with the probability to form a second hole characterized by the quantum numbers "p". One has to replace  $\langle A'_{b'} | a_i | G \rangle$  by  $\langle A'_{b',p';q'} | a_i | G \rangle$  with the two electron holes  $b'$  and  $p'$  and the additional electron particle  $q'$  in Dysprosium above the Fermi surface  $F$ . These expressions are derived in reference [7]. The formulas for three-hole states can be obtained with the help of Wick's theorem [25] from:

$$P_{p'_1, p'_2 / b'} = |\langle A'_{p'_1, p'_2, b'} | a_b | G \rangle|^2 = \sum_{q'_1, q'_2 > F} |\langle 0 | a'_{q'_2} a'_{q'_1} a'_{Z'=Z-1} \dots a'_{p'_2+1} a'_{p'_2-1} \dots a'_{p'_1+1} a'_{p'_1-1} \dots a'_{b'+1} a'_{b'-1} \dots a'_1 a_b a_1^\dagger a_2^\dagger a_3^\dagger \dots a_Z^\dagger | 0 \rangle|^2 \quad (5)$$

With the Holmium ground state:

$$|G\rangle = a_1^\dagger a_2^\dagger a_3^\dagger \dots a_Z^\dagger |0\rangle \quad (6)$$

The probability for forming three hole states can be calculated in the sudden approximation with the help of Wick's theorem [25].

To evaluate the probability for two electron particle-hole states one sums incoherently over all unoccupied states  $q'_1$  and  $q'_2$ .

$$P_{b', p_1, p_2} = \sum_{q'_1 < q'_2 < F} |\langle p_{1 < F, Ho} | q'_{1 > F, Dy} \rangle \langle q'_{1 > F, Dy} | p_{1 < F, Ho} \rangle \cdot \langle p_{2 < F, Ho} | q'_{2 > F, Dy} \rangle \langle q'_{2 > F, Dy} | p_{2 < F, Ho} \rangle| \cdot \prod_{k=k' < F_{Dy} \neq b, p_1, p_2} |\langle k'_{Dy} | k_{Ho} \rangle|^2 \quad (7)$$

Here as stressed above the sum over  $q'_1$  and  $q'_2$  runs over the unoccupied bound and continuum states in Dy. The dependence on the quantum numbers  $b = b'$  originates from the fact, that the electron orbitals in Dysprosium are calculated selfconsistently with an empty state  $b'$ . One can now use the

completeness relation to shift the sum over  $q'_i$  from the continuum to states, which one can calculate easier. One divides the completeness relation into two pieces: up to the last occupied state below the Fermi Surface F and all states above the last occupied state including also the continuum.

$$1 = \sum_{q'_i < F} \langle p_i | q'_i \rangle \langle q'_i | p_i \rangle + \sum_{q'_i > F} \langle p_i | q'_i \rangle \langle q'_i | p_i \rangle$$

with:  $i = 1 \text{ and } 2$

(8)

The sum in eq. (7) is the last part of the completeness relation (8) and one can transcribe (7) into:

$$P_{p_1, p_2/b} = \left( 1 - \sum_{q'_1 < F} \langle p_{1, Ho} | q'_{1, Dy} \rangle \langle q'_{1, Dy} | p_{1, Ho} \rangle \right) \cdot \left( 1 - \sum_{q'_2 < F} \langle p_{2, Ho} | q'_{2, Dy} \rangle \langle q'_{2, Dy} | p_{2, Ho} \rangle \right) \cdot \prod_{k=k' < F_{Dy}; \neq p_1, p_2, b} \langle k'_{Dy} | k_{Ho} \rangle$$
(9)

An interchange of two electrons from the states  $b'$ ,  $p'_1$  and  $p'_2$  yields due to the antisymmetrization a "-" sign. But for the probability the minus sign is irrelevant. In the literature one uses often the Vatai approximation [23, 24]: Exchange corrections have been neglected already in eq. (9). In addition one assumes, that the overlaps of electron wave functions in the parent and the daughter atom with the same quantum numbers can be approximated by unity. Typically the overlaps have values [5]  $\langle k'_{Dy} | k_{Ho} \rangle \approx 0.999$  and  $0.999^{65} \approx 0.94$ . In the Vatai approximation [24] one replaces this value by 1.0. The probability for two one-electron particle and one-electron hole states is the product. The probability for a second hole in  $p'_i$  with a first hole in  $b'$  is:

$$P_{p_i/b'} = \left( 1 - \sum_{q'_i < F} \langle p_{i, Ho} | q'_{i, Dy} \rangle \langle q'_{i, Dy} | p_{i, Ho} \rangle \right) = \left( 1 - \langle p_{i, Ho} | p'_{i, Dy} \rangle \langle p'_{i, Dy} | p_{i, Ho} \rangle - \sum_{q' < F, \neq p'_i} \langle p_{i, Ho} | q'_{i, Dy} \rangle \langle q'_{i, Dy} | p_{i, Ho} \rangle \right)$$
(10)

The physics of the two terms subtracted from 1 in eq. (10) is: The first subtracted term gives the probability, that the state  $p'_i$  in Dy is occupied. The sum in the second term takes into account the Pauli principle and prevents, that electrons can be moved into occupied states in Dy. The single electron states like  $|p\rangle = |n, \ell, j, m\rangle$  include also the angular momentum projection quantum number m. This projection is for the description of the data irrelevant. The first subtracted term in eq. (10) gives the probability, that a specific magnetic substate  $m'_i$  is occupied in  $p'_i$ . The probability, that all magnetic substates of  $p'_i$  are occupied, is the product of the probabilities of all substates. One obtains the Nth. power of the single electron probability with  $N_{p'_i} = N_{n, \ell, j; p'_i} = (2j+1)_{p'_i}$ :

$$| \langle (n, \ell, j)_{p_i, Ho} | (n, \ell, j)_{p'_i, Dy} \rangle |^{2N_{n, \ell, j; p'_i}}.$$

$$P_{p_i/b} = (1 - | \langle (n, \ell, j)_{p_i, Ho} | (n, \ell, j)_{p'_i, Dy} \rangle |^{2N_{p'_i}} - \sum_{(n, \ell, j), q'_i, Dy; \neq p'_i < F} \frac{N_{n, \ell, j} N_{n', \ell, j}}{2j+1} | \langle (n, \ell, j)_{p_i, Ho} | (n, \ell, j)_{q'_i, Dy} \rangle |^2)$$
(11)

With the expression:

$$\text{for } n \text{ and } n' \text{ with: } N_{n, \ell, j} = N_{n', \ell, j} = 2j+1;$$
(12)

For the hole state in b' and the partially occupied state in  $4f_{7/2'}$  one needs special expressions:

$$N_{(n,\ell,j)_b} = 2j_b \quad \text{and} \quad N_{4f_{7/2'}} = 5. \quad (13)$$

Here  $|b\rangle$  is the orbital in Ho, from which the electron is originally captured. The electron  $p'_i$  is moved to  $4f_{7/2}$  with now 5 electrons in this orbit. Thus the second (and the third) hole is in Dy in the  $p'_i$  electron orbit.  $N_{n,\ell,j}/(2j+1)$  is the averaged probability to find an electron in the  $|p, n, \ell, j, m\rangle$  orbital and  $N_{n',\ell,j}$  the number of electron in the  $|q'_i, n', \ell, j\rangle$  state. Primes always indicate states in Dy. For the probability of the second and third hole we used the so called Vatai [23, 24] approximation with the overlaps of corresponding electron wave function with the same quantum numbers in Ho and Dy equal to unity and neglected the exchange corrections. For the "diagonal" overlaps of the order of  $\approx 0.999$  this is a good approximation.

For the probability of two different electron-particle electron-hole excitations  $q_1(p_1)^{-1}$  and  $q_2(p_2)^{-1}$  in eq. (7), eq. (9) one has now to introduce eq. (10) or finally eq. (11) as product for particle 1 and particle 2 into eq. (9).

$$\begin{aligned} P_{p_1, p_2/b} &= P_{p_1/b} P_{p_2/b} \prod_{k=k' < F_{Dy}; \neq p_1, p_2, b} < k'_{Dy} | k_{Ho} > \approx \\ &\quad (1 - | < (n, \ell, j)_{p_1, Ho} | (n, \ell, j)_{p'_1, Dy} > |^{2N_{p'_1}} - \\ &\quad \sum_{(n, \ell, j), q'_1, Dy; \neq p'_1 < F} \frac{N_{n, \ell, j, p_1, Ho} N_{n', \ell, j, q_1, Dy}}{2j+1} | < (n, \ell, j)_{p_1, Ho} | (n, \ell, j)_{q'_1, Dy} > |^2) \\ &\quad (1 - | < (n, \ell, j)_{p_2, Ho} | (n, \ell, j)_{p'_2, Dy} > |^{2N_{p'_2}} - \\ &\quad \sum_{(n, \ell, j), q'_2, Dy; \neq p'_2 < F} \frac{N_{n, \ell, j, p_2, Ho} N_{n', \ell, j, q_2, Dy}}{2j+1} | < (n, \ell, j)_{p_2, Ho} | (n, \ell, j)_{q'_2, Dy} > |^2) \\ &\quad \prod_{k=k' < F_{Dy}; \neq p_1, p_2, b} < k'_{Dy} | k_{Ho} > \end{aligned} \quad (14)$$

The three hole probability normalized to the electron capture from the M1  $3s_{1/2}$  state in 163 Holmium is then obtained by multiplying this expression into eq. (4) for  $P_b$ .

### 3 Spectrum of Dy deexcitation with three-hole states.

Tables 1 and 2 show the one and the two-hole resonance energies and widths taken from results of the ECHo collaboration [16, 19] and, if there not available, taken in reference [6] from the handbook of Chemistry tabulated by Weast [26]. The two hole energies are determined as the one hole excitation energy in Dysprosium and the electron-particle and electron-hole energy differences in Holmium. Due to the hole in an inner shell in Dysprosium, which reduces the shielding of the charge of the nucleus, the field looks for the outer electron particle-hole excitation in Dysprosium similar as in Holmium with one positive charge more. This is only a rough approximation, since the relevant one-hole states in Dysprosium M1  $3s_{1/2}$ , N2  $3p_{1/2}$ , N1  $4s_{1/2}$ , N2  $4p_{1/2}$ , O1  $5s_{1/2}$  and O2  $5p_{1/2}$  are not from the inner most shells and thus the outer particle-hole states in Dysprosium do feel only approximately the selfconsistent field of Holmium. A better approach would be to calculate the electron particle-hole energies in Dysprosium selfconsistently with the hole in the appropriate state. This could shift the resonance energies by a few eV compared to the approximation used.

The three hole resonance energy is in the approximation discussed:

Table 1: The 6 one-hole excitation energies  $E_C$  and the widths  $\Gamma$  in  $^{163}\text{Dy}$  with quantum numbers  $n, \ell, j$  are listed according to the ECHo collaboration [13, 16, 19, 20] as far as they are available. Else they are taken according to ref. [6] from Weast tabulated in the Handbook of Chemistry [26], although there seem to be in some cases better values in the literature [29], [30], [31] and [32]. A Q value of 2.8 keV allows energetically the excitations of the one hole states: M1 3s1/2, M2 3p1/2, N1 4s1/2, N2 4p1/2, O1 5s1/2 and O2 5p1/2. Electrons can be captured only from s1/2 and p1/2 orbits in Holmium, since other electrons have no probability to be at the nucleus.

<i>Number</i>	<i>1. hole</i>	$E_c[\text{eV}]$	$\Gamma[\text{eV}]$	<i>Number</i>	<i>1. hole</i>	$E_C$	$\Gamma[\text{eV}]$
1	3s1/2	2040	13.7	2	3p1/2	1836	7.2
3	4s1/2	411	5.3	4	4p1/2	333	8
5	5s1/2	48	4.3	6	5p1/2	30.8	3

$$E_{c,(b,p_1,p_2)^{-1}} = E_{c,b^{-1},Dy} + (E_{c,(p_1)^{-1},Ho} - E_{c,q_1,Ho}) + (E_{c,(p_2)^{-1},Ho} - E_{c,q_2,Ho}) \quad (15)$$

Where the particle-hole energy can be taken from table 2 starting with column four and subtracting the corresponding one hole energy  $E_{c,b^{-1}}$  from table 1:

$$E_{c,particle-hole \text{ energy}} = (E_{c,(p_1)^{-1},Ho} - E_{c,q_1,Ho}) = E_{c,(b,p_1)^{-1}}(\text{column 4, table 2}) - E_{c,b^{-1}}(\text{one - hole energy of table 1}) \quad (16)$$

The numerical calculation includes 6 Dysprosium one-hole states 3s1/2, 3p1/2, 4s1/2, 4p1/2, 5s1/2 and 5p1/2 listed in table 1 and 39 two-hole states tabulated in table 2. The problem of choosing for the particle-hole energies the values of Ho is discussed before eq. (15). The allowed 208 three hole states in this work are limited by energy conservation to  $Q \geq E_C(1-hole) + E_C(particle-hole_1) + E_C(particle-hole_2)$  with the particle-hole excitation energies  $E_C(particle-hole)$ , taken as the ionization energy in Ho.

Figure 1 shows the sum of the one-, two- and three-hole theoretical bolometer spectrum after electron capture in Holmium for the deexcitation of Dysprosium. The electron wave functions are calculated in a Dirac-Hartree-Fock approach [8, 9, 10]. In Dysprosium the one-hole states are included in the selfconsistent determination of the electron wave functions. A detailed description of our approach is given in ref. [7]. Figure 2 compares the sum of the theoretical one- plus two- plus three-hole deexcitation spectrum of Dysprosium with the results including only one-hole states and the sum of the one- and the two-hole states. The three-hole contributions can practically not be seen even in this *logarithmic*<sub>10</sub> plot.

Figure 3 compares in a *logarithmic*<sub>10</sub> plot the deexcitation spectrum of the one-hole states, the one- plus two-hole and the one- plus two- plus three-hole excitations with the ECHo data [19, 20]. The experimental spectrum is binned into 2 eV. Since some bins contain no counts the logarithmic value is  $-\infty$ . For a more relevant logarithmic plot one would need more counts. The three experimental peaks between 1.2 and 1.6 keV originate from a  $^{144}\text{Pm}$  contamination. Figure 4 shows in detail a comparison of the one-, the two- and three-hole spectrum with the ECHo data [19, 20] around the area of the N2 4p1/2, 0.333 keV and the N1 4s1/2, 411 keV hole states.

Table 2: The 39 (from number 7 to 45) two-hole excitation energies  $E_C$  and the widths  $\Gamma$  in  $^{163}\text{Dy}$  with quantum numbers  $n, \ell, j$  are listed. We adopt here the first hole values for  $E_c$  and the width  $\Gamma$  of the ECHo collaboration [13, 16, 19, 20] and add the excitation energies of the second hole taken by Robertson [6] taken from Weast tabulated in the Handbook of Chemistry [26]. The data measured by the ECHo collaboration [19], [20] are listed in table 1. The energies of the two hole states are given by  $E_C(1\text{holeDy}) + E_C(\text{particle} - \text{hole excitation in Ho})$ . The number of one- and two-hole states listed in tables 1 and 2 is 45. The allowed 208 three hole states included in this work are limited by energy conservation to  $Q \geq E_C(1 - \text{hole}) + E_C(\text{particle} - \text{hole}_1) + E_C(\text{particle} - \text{hole}_2)$ . The particle-hole excitation energies  $E_C(\text{particle} - \text{hole})$  can be obtained from this table by subtracting the one-hole excitations given in table 1.

<i>Number</i>	1. hole	2. hole	$E_c[\text{eV}]$	$\Gamma[\text{eV}]$	<i>Number</i>	1. hole	2. hole	$E_C$	$\Gamma[\text{eV}]$
7	3s1/2	4s1/2	2472.4	13.7	27	4s1/2	4s1/2	841.4	5.4
8	3s1/2	4p1/2	2385.3	13.2	28	4s1/2	4p1/2	752.5	5.4
9	3s1/2	4p3/2	2350.0	13.2	29	4s1/2	4p3/2	717.2	5.4
10	3s1/2	4d3/2	2201.8	13.2	30	4s1/2	4d3/2	569.0	5.4
11	3s1/2	4d5/2	2201.8	13.2	31	4s1/2	4d5/2	569.0	5.4
12	3s1/2	4f5/2	2050.4	13.2	32	4s1/2	4f5/2	417.6	5.4
13	3s1/2	4f7/2	2047.0	13.2	33	4s1/2	4f7/2	414.2	5.4
14	3s1/2	5s1/2	2091.1	13.2	34	4s1/2	5s1/2	458.3	5.4
15	3s1/2	5p1/2	2072.6	13.2	35	4s1/2	5p1/2	439.8	5.4
16	3s1/2	5p3/2	2065.9	13.2	36	4s1/2	5p3/2	433.1	5.4
17	3p1/2	4s1/2	2269.2	6	37	4p1/2	4p1/2	671.8	5.3
18	3p1/2	4p1/2	2180.3	6	38	4p1/2	4p3/2	636.5	5.3
19	3p1/2	4p3/2	2145.0	6	39	4p1/2	4d3/2	488.3	5.3
20	3p1/2	4d3/2	1996.8	6	40	4p1/2	4d5/2	488.3	5.3
21	3p1/2	4d5/2	1996.8	6	41	4p1/2	4f5/2	336.9	5.3
22	3p1/2	4f5/2	1845.4	6	42	4p1/2	4f7/2	328.3	5.3
23	3p1/2	4f7/2	1842.0	6	43	4p1/2	5s1/2	377.6	5.3
24	3p1/2	5s1/2	1886.1	6	44	4p1/2	5p1/2	359.1	5.3
25	3p1/2	5p1/2	1887.6	6	45	p1/2	5p3/2	352.4	5.3
26	3p1/2	5p3/2	1860.9	6	—	—	—	—	—

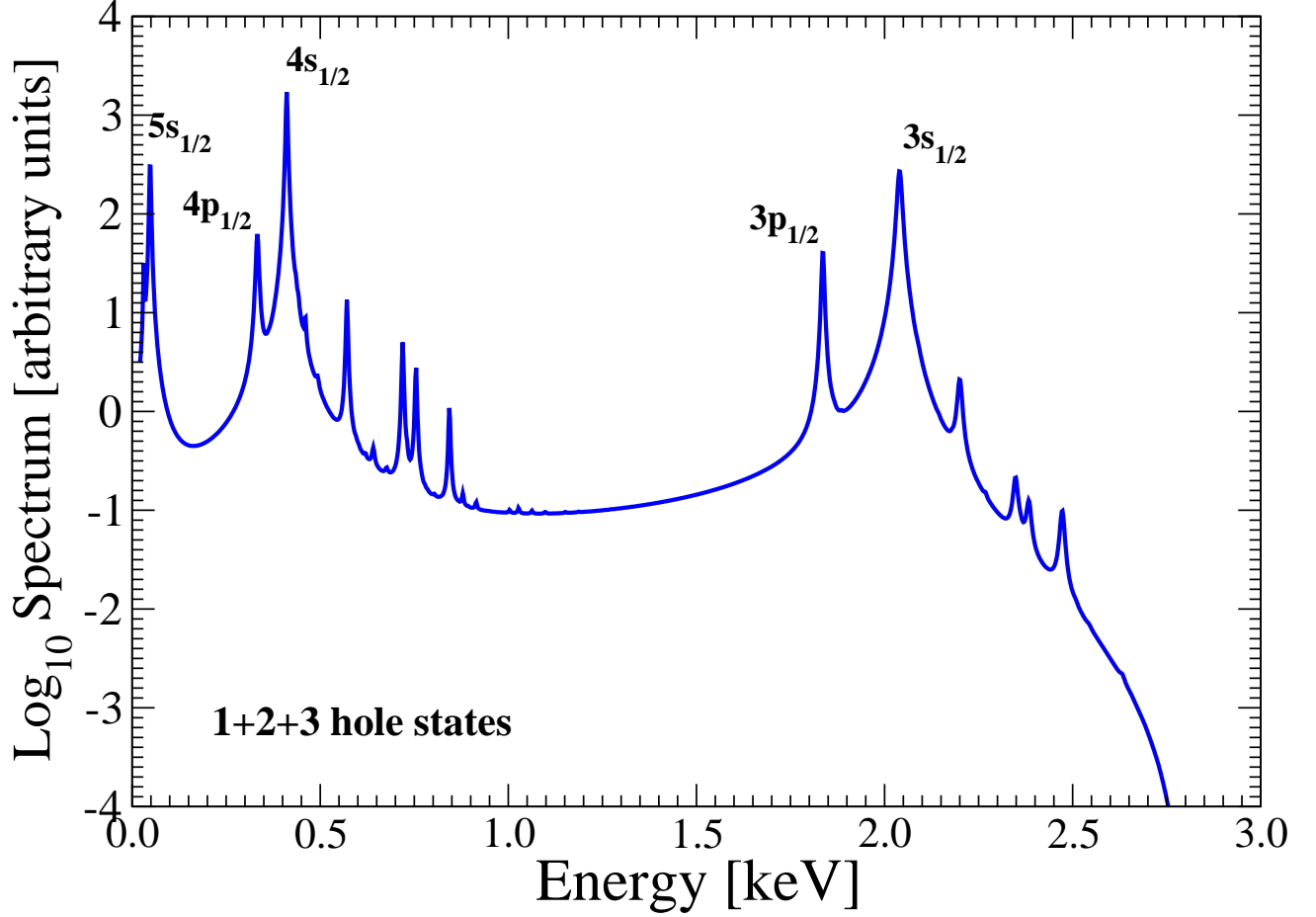


Figure 1: (Color online) *Logarithmic*<sub>10</sub> three-hole Bolometer Spectrum (2) with (4) and (14) for the sum of the one-, the two- and the three-hole probabilities calculated in this work with the resonance energies and widths of tables 1 and 2 with the assumed Q-value  $Q = 2.8$  keV for the bolometer energy between 0.0 and 2.8 keV. The *logarithmic*<sub>10</sub> coordinates of the ordinate has to be read as  $10^{\text{ordinate}}$ , e. g. the ordinate value -2 corresponds to  $10^{-2}$ . The theoretical spectra are normalized to the experimental  $4s_{1/2}$  hole peak at 0.411 keV (see figure 4).



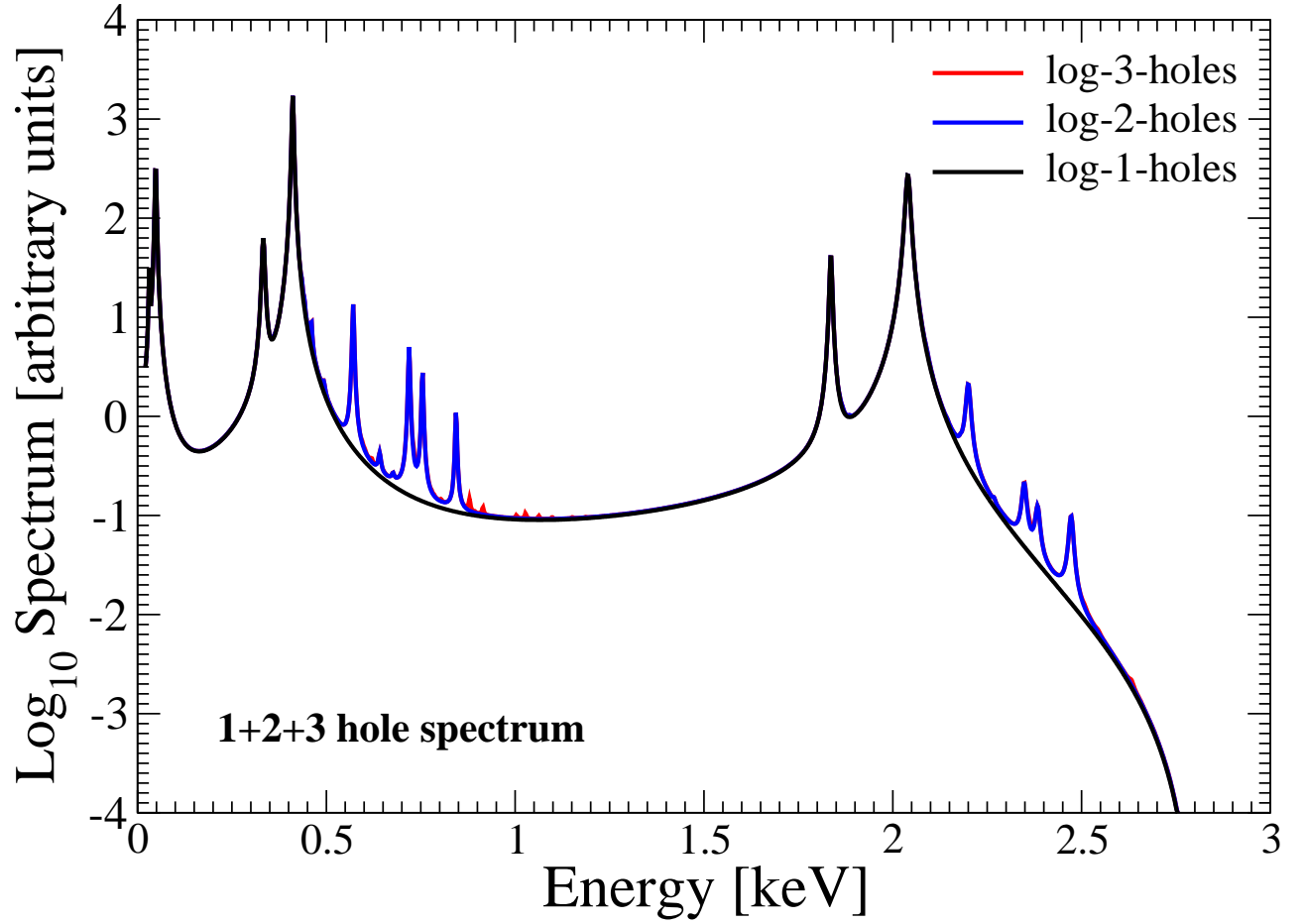


Figure 2: (Color online) *Logarithmic*<sub>10</sub> plot of the bolometer spectrum (2) including the one-, the one- plus two- and the one- plus two- plus three-hole states. The Q value is assumed to be  $Q = 2.8$  keV according to an ECHo measurement [16]. The resonance energies and widths are listed in tables 1 and 2. The three-hole states can hardly be seen.

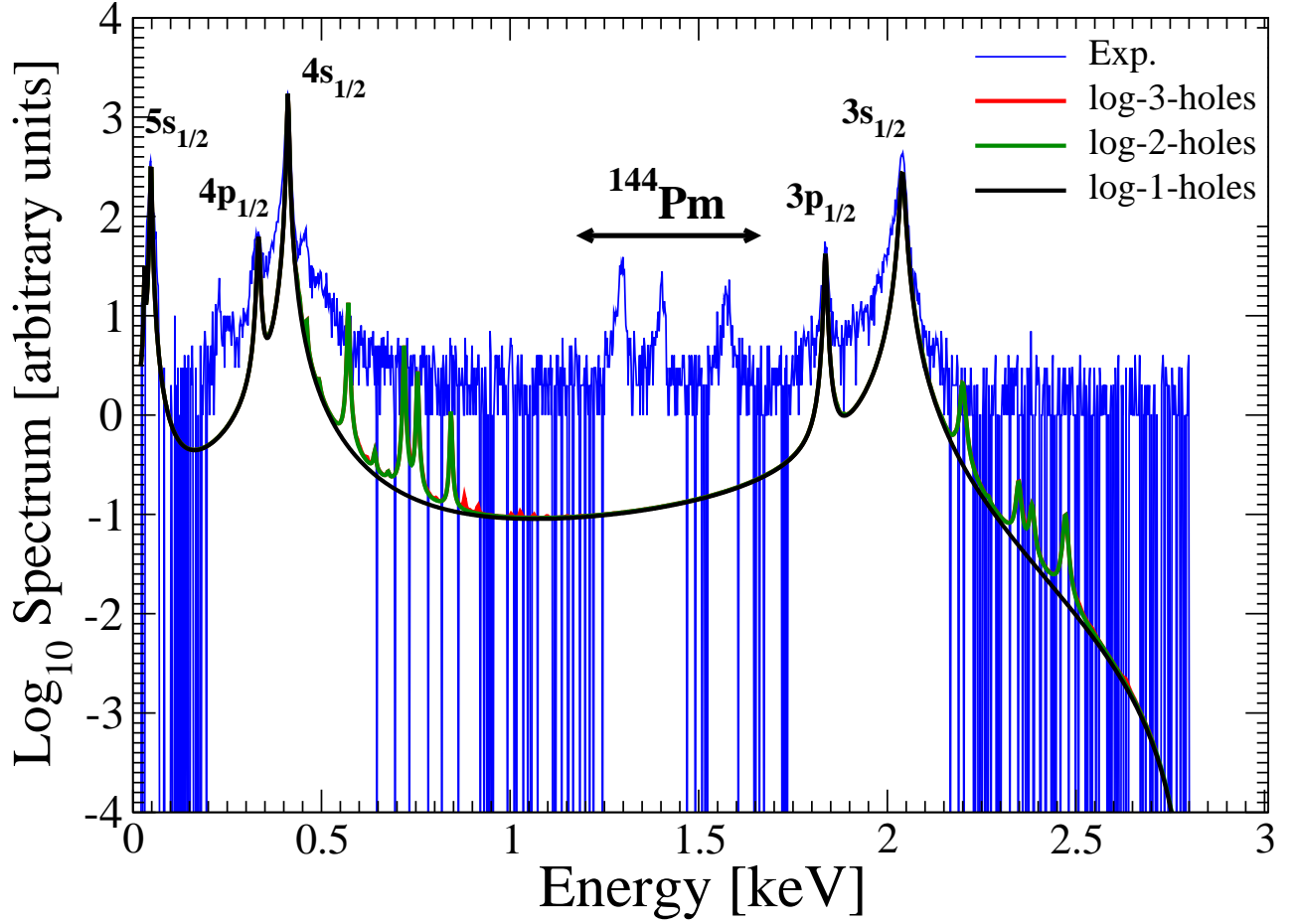


Figure 3: (Color online) Comparison of the *logarithmic*<sub>10</sub> spectrum of the one-, the one- plus two and the one- plus two- plus three-hole states with the data of the ECHo collaboration [13, 16, 19, 20]. The experimental data are binned in 2 eV (counts/2eV). Some bins have no counts and thus the logarithmic value is  $-\infty$ . The effect of the three-hole states does show hardly up even in this logarithmic figure.

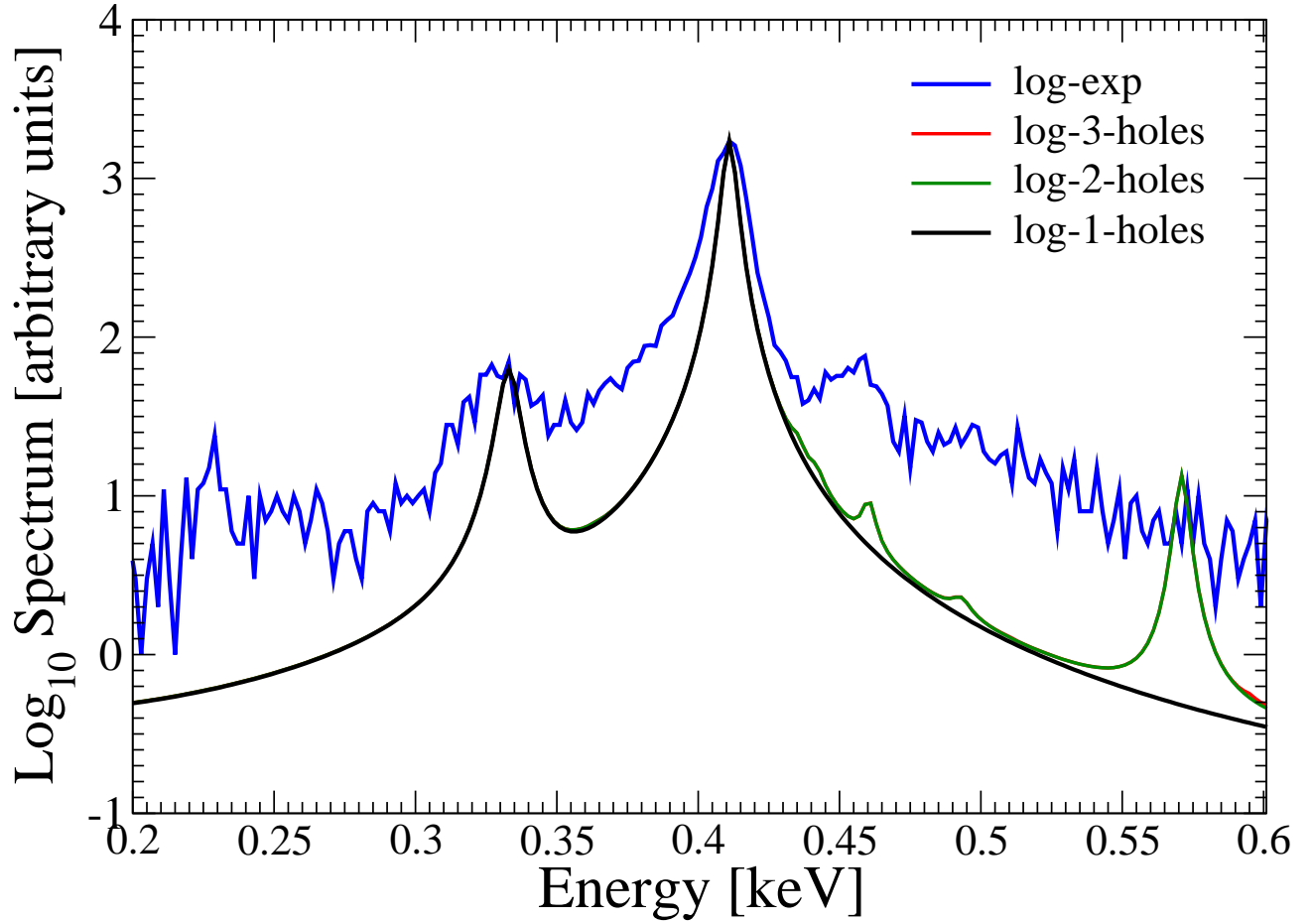


Figure 4: (Color online) Comparison of the *logarithmic*<sub>10</sub> bolometer spectrum around the  $N1 = (4s1/2)^{-1}$  0.411 keV (relative probability to the 3s1/2 state 24.4 %) and  $N2 = (4p1/2)^{-1}$  0.333 keV (relative probability 1.22 %) resonances with the ECHo data [13, 16, 19, 20]. At 0.569 keV one sees the degenerate two-hole states  $N1 = (4s1/2)^{-1}$ ,  $N4 = (4d3/2)^{-1}$  with the relative probability of 0.088 % and  $N1 = (4s1/2)^{-1}$ ,  $N5 = (4d5/2)^{-1}$  with the relative probability of 0.125 % (total 0.213 %). The resonance energies and the widths are taken from the ECHo collaboration [19, 20]. The theory does not include the finite resolution of the measurement, which is about 10 eV FWHM. The sum of the one- plus two- and the sum of the one- plus two- plus three-hole spectra look even in a logarithmic scale identical.

## 4 Conclusions

Here for the first time the effect of three hole excitations on the deexcitation spectrum of  $^{163}\text{Dy}$  after electron capture in  $^{163}\text{Ho}$  has been studied. The spectrum is not affected visibly by the three-hole excitations of electron hole and two electron particle-hole states. The two electron particle-hole excitations plus the energy of a one-hole state (three-hole states) add up to higher energies as the one- and two-hole states. A one-, two- or three-hole state close to the Q value could complicate the determination of the neutrino mass. If only one resonance state is dominant near the Q value and if the line profile is Lorentzian (or of an other known analytical form with two parameters), one needs to fit simultaneously four parameters of the theory to the data: (1) neutrino mass, (2) the energy difference of the dominant state to the Q value, (3) the width of the resonance and (4) the strength. Before fitting the four parameters of the theoretical spectrum to the data, one has to fold the experimental spectral function of the detector into the theoretical results. The situation is more complicated, if several one- or two- or three-hole resonances contribute to the shape of the spectrum near the Q value. With the recent measurement [19, 20] of the ECHo collaboration (3)  $Q = 2.80 \pm 0.08 [\text{keV}]$ , it has been shown by Faessler and Simkovic [7], that the highest one hole state is dominant at the Q value. The high energy two-hole states do not influence the behavior in the region, which determines the neutrino mass [7]. Although the three-hole excitations can come closer to the Q value, their strength is so weak, that they are not dangerous for the determination of the neutrino mass.

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